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Nature of Buoyancy-Driven Flows in a Reduced-Gravity Environment

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The role of buoyancy-driven convection in reduced-gravity environments has been emphasized. It is shown that for some materials-processing experiments the values of the fundamental dimensionless parameters such as *Gr* or *Ra* are shifted from the very large (ground-based values) to moderately large (space-based values). As a consequence, in cases where approximate analytical solutions are desired, the accuracy of the usual asymptotic (boundary-layer) analysis in which infinitely large parametric ranges are assumed is reduced. Approximate analytical techniques for refinement of the asymptotic solutions are identified that will extend the accuracy to the moderately large parametric ranges associated with space-based processing.

Introduction

AN understanding of the effect of gravity on fundamental fluid physics and transport phenomena is of interest for both practical and basic research reasons. In a reduced-gravity environment, various types of convection have been identified.[1] Among them are surface tension induced convection, thermosolutal convection, phase-change convection, and buoyancy-induced convection.

In a spacecraft the buoyancy-driven convection can take place as a result of both constant and varying accelerations of different levels.[2] Steady g levels can be caused by gravity gradients, solar pressure, and spacecraft rotation, whereas varying g levels result from such things as vibrations from machinery or astronaut movement. Although, in general, the bouyant convection encountered aboard a spacecraft is several orders of magnitude lower than under similar conditions on the ground, it is by no means negligible or unimportant.[1,3] To fully understand the impact of reduced but nonzero gravity on buoyant convection, any change in the level and direction of gravity must be related to corresponding variations of the fundamental dimensionless group parameters of the particular process. It will be shown later than not only will a reduction in g level fail to alleviate totally any undesirable convection flow but may even shift the "large" values of corresponding parameters such as Gr or Ra (as typical for most ground-based materials-processing experiments) into an "intermediate" range for which approximate analytical solutions are most difficult to obtain. As a consequence, a true understanding of the problem and considerable physical insight, which is in general a byproduct of closed-form analytical solutions, is no longer readily possible. The purpose of this paper is to emphasize the role of buoyancy-driven convection in low-gravity environments and to discuss analytical treatment of the corresponding natural convection problems. Other important and often dominant forms of convection, such as surface tension induced convection, acoustic convection, and phase-change convection, have been considered by many researchers and are not discussed herein.

General Considerations

Buoyancy-driven convection arises because of a density variation caused by heat and mass transfer processes in a body force field such as a gravitational field. If there exists a simultaneous presence of temperature and/or concentration gradients, the resulting phenomena are called thermosolutal convection. This type of convection is frequently encountered in materials processing on Earth and in a modified form (as determined by changes in fundamental parameters) in space-based processing.

As pointed out by both Ostrach[1] and Siegel,[3] the important parameters in natural convection are

1) the Grashof number

$$Gr = \frac{\beta g \Delta T \ell^3}{\nu^2}$$

2) the Prandtl number

$$Pr = \frac{\nu}{\alpha}$$

where α is the thermal diffusivity, and

3) the Rayleigh number that is defined as

$$Ra = Pr \cdot Gr$$

The Rayleigh number gives a measure of the ratio of the heat convected by a fluid to that conducted through the fluid.

Both the Rayleigh number and the Grashof number are directly proportional to acceleration due to gravity g and, hence, a reduction in g as compared with its value on Earth, g_0 will reduce Ra and Gr by the same amount.

The attainable g levels in various reduced-gravity facilities are influenced by absolute rotation and angular accelerations of the platform, forces acting on it, and nonuniformities of the external force fields. The time duration of the reduced-gravity environment is also dependent on the facilities, i.e., satellites, orbiting space stations, rockets, aircraft, and the drop tower. Figure 1 from Ref. 4 represents graphical data relating the achievable nominal gravitational levels as function of durations with the available microgravity facilities where

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the intensity, direction, and duration of occasional disturbances define the "quality" of the platform's nominal gravitational level. As shown in Fig. 1, while very low gravitational levels can be attained in the drop tower facilities (a mean g on the order of $10^{-6}g_0$), the relative duration of each run is on the order of a few seconds. On the other hand, if time durations on the order of several hours are required, as typically is the case in many materials-processing applications, only mean nominal gravitational levels of $10^{-3}g_0$ to $10^{-4}g_0$ are achievable with existing facilities. Many investigators[5] consider an average gravitational level of $10^{-3}g_0$ to represent a readily achievable long duration reduced-gravity environment. Determination of accurate values of gravitational levels is the subject of current investigations.[4]

Now let's consider some typical examples of space processes. A range of Rayleigh numbers encountered in reduced-gravity environments is presented in Ref. 3 for air, water, and liquid hydrogen. A modified Rayleigh number ($Ra_x^* = g\beta q x^4/\alpha\nu$ where q is the wall heat flux) is defined to describe the effects of heat flux imposed by solar radiation on convection of the fuel inside the storage tank. Figure 2 from Ref. 3 shows Ra_x^* as a function of g/g_0 for a characteristic length dimension

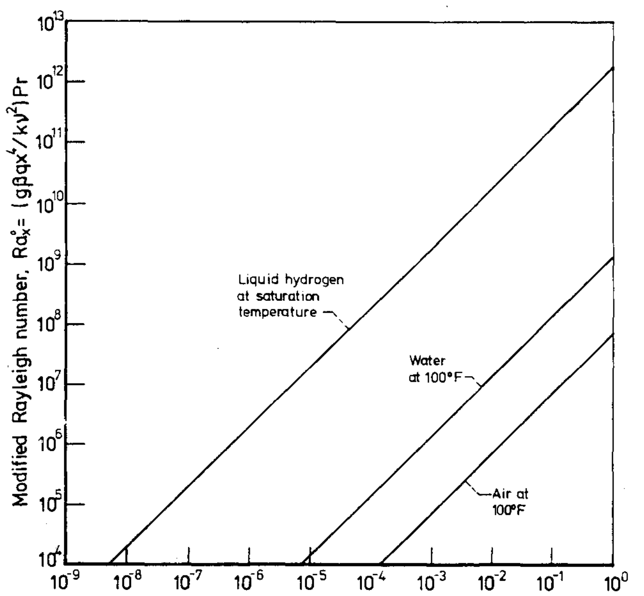

Fig. 1 Nominal gravitational levels as a function of durations achievable with the main available microgravity platforms.[4]

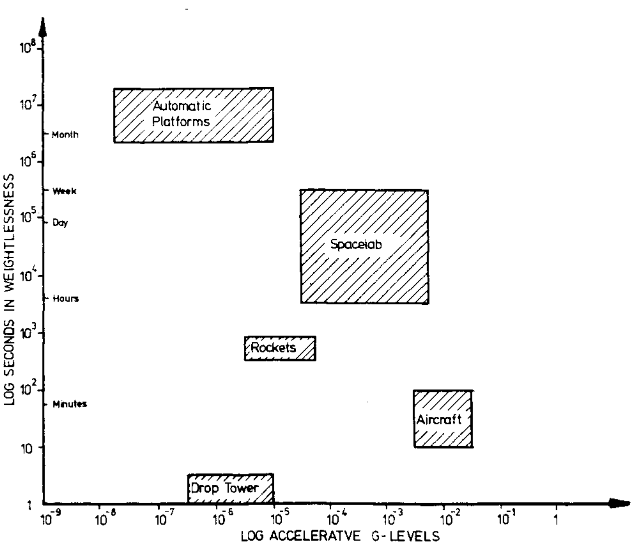

Fig. 2 Fraction of Earth gravity, g/g_e.[3]

of 1 ft and for a wall heat flux of 1 Btu/h/ft². It is interesting to note that, for very low gravitational levels of $10^{-6} \le g/g_0 \le 10^{-3}$, the corresponding Rayleigh numbers are on the order of 10^4–10^9. Since the Rayleigh number is a measure of the ratio of the heat convected by a fluid to that conducted through it, a large Ra clearly indicates the importance of buoyancy-driven convection for these conditions that are typically encountered for various space applications.

Reference 1 gives examples of calculated dimensionless parameters for several processes. The calculations show that under the most mundane conditions, e.g., temperature differences 10°C at levels of about 20°C and a characteristic length of 10 cm, the Grashof number at 1 g for air is on the order of 10^6, for water is 10^7, and for liquid metals is about 10^9. Considering a mean reduced g level on the order of 10^{-4}–10^{-3}, it can be seen from Table 1 that the Grashof numbers are still large and considerable buoyant convection can occur. Recently, dimensionless parameters for some typical crystal growth processes representing alloys of Germanium have been tabulated.[6-8] The calculated values of Grashof numbers for each case are listed in Table 1. Note that the corresponding Grashof numbers for a reduced-gravity environment can still reach relatively high values, indicating a relatively high characteristic velocity resulting from buoyant convection for each process.

Remarks on Approximate Analysis in Low g Problems

Many configurations related to material processing are ones in which the fluid is confined by rigid boundaries. A rectangular enclosure, generally having either high or low aspect ratios, has been chosen by many investigators to represent a typical container configuration for basic research purposes.[9-11] The container may contain fluids with Prandtl numbers ranging from very low, as appropriate for liquid metals, to very high, for very viscous fluids such as molten glasses. Various combinations of thermal and/or solutal boundary conditions are possible from which the low aspect ratio enclosures with insulated horizontal walls and differentially heated vertical walls have been given particular consideration. This configuration is closely related to many crystal growth processes such as chemical vapor transport, solution crystal growth, and crystal growth from melts.[11,12] It must be mentioned, however, that the nature of the flow and transport processes for other geometric configurations and for different orientation of density gradients with respect to body force direction might be fundamentally different,[13] and each particular configuration requires independent consideration. To give a quantitative example of the previous discussion, consider the problem of high Rayleigh number convection in rectangular enclosure with Prandtl number taken on the order of unity. Researchers have extensively used the configuration because of its simplicity and overall representation of many real process.[9,14-18] The opposing vertical walls are maintained at hot and cold temperatures, and the horizontal walls are insulated. Unfortunately, as was pointed out earlier, analytical treatment for this category of confined natural convection problems is most difficult, in particular for cases with high Grashof numbers where both nonlinear convection and inertial terms are of significance and

Table 1

Processes identified by reference numbers		Grashof numbers based on g_0	Grashof numbers based on $g = 10^{-3}\ g_0$
	Air	10^6	10^3
Ref. 1	Water	10^7	10^4
	Liquid	10^9	10^6
	Air	1.4×10^8	1.4×10^5
Ref. 3	Water	1.7×10^8	1.7×10^5
	Liquid Hydrogen	10^{12}	10^9
Ref. 6	Gall. doped Germ	0–4.3×10^{10}	0–4.3×10^7
Ref. 7	Binary AL. of Germ	0–7×10^8	0–7×10^5

must be fully dealt with (for a special case of very large Prandtl numbers and small or unit order Grashof numbers, the inertial terms may be neglected due to highly viscous nature of the process).

An intensively used analytical method for solving high Rayleigh number, confined natural convection problems is to decompose the governing equation into the boundary-layer and core equations. Solutions for the core and boundary-layer equations can then be obtained separately, provided they match at an overlap region. A more detailed discussion and a comprehensive review of natural convection heat transfer in cavities and cells can be found in Refs. 9, 19, and 20. It is important to note that any asymptotic solution obtained in this manner is, strictly speaking, an adequate solution for Gr (or Ra)$\rightarrow\infty$; i.e., the solution is appropriate for extremely large values of Gr (or Ra), say, 10^8 or 10^9. One only hopes that the solution remains valid for moderately large Grashof (Rayleigh) numbers on the order of 10^3–10^6. As will be seen later, there are possible improvements that could be performed to upgrade the asymptotic solutions and therefore validate them for a wider range of Grashof (Rayleigh) numbers. The foregoing discussion becomes most relevant when one notices that the effect of reducing the gravity level by an order of 10^{-4} to 10^{-3} is equivalent to reducing the Grashof (Rayleigh) numbers from a high value of 10^9 for a certain process to a more moderate range of 10^5 or 10^6 for the same process. Consequently, if a boundary layer solution is obtained for $Ra \sim 10^9$, it will not be valid, in general, for moderately large $Ra \sim 10^5$ or 10^6 unless additional care is taken to upgrade the asymptotic solution and therefore increase its range of validity. Consider the high Rayleigh number convection in a rectangular enclosure with differentially heated vertical walls and aspect ratio $\epsilon \leq 1$ (Fig. 3). A recent experimental study by Kamotani et al.[14] investigated the dependence of the Nusselt number (dimensionless heat transfer coefficient) on the Rayleigh number and the enclosure's aspect ratio. Figure 4 shows the correlation for

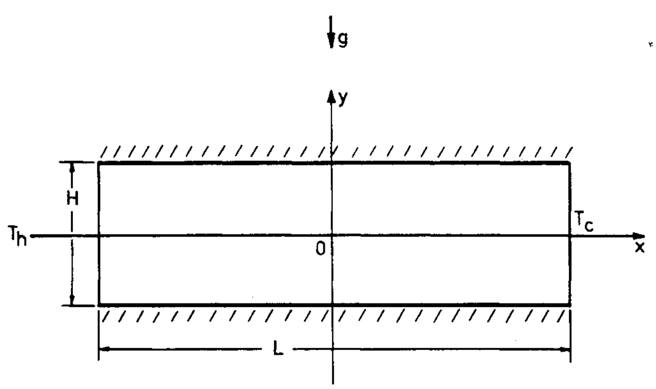

Fig. 3 Physical system.

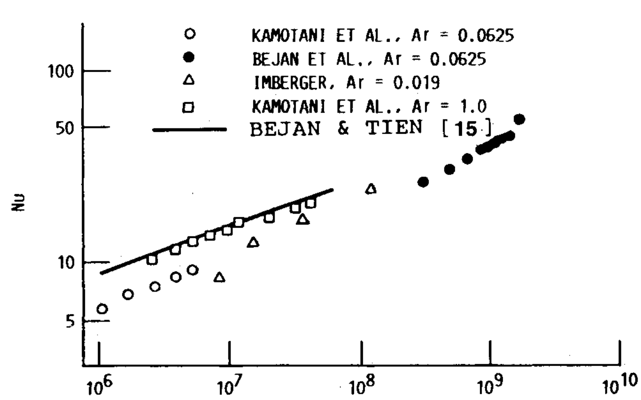

Fig. 4 Nusselt number in the boundary-layer regime.[15]

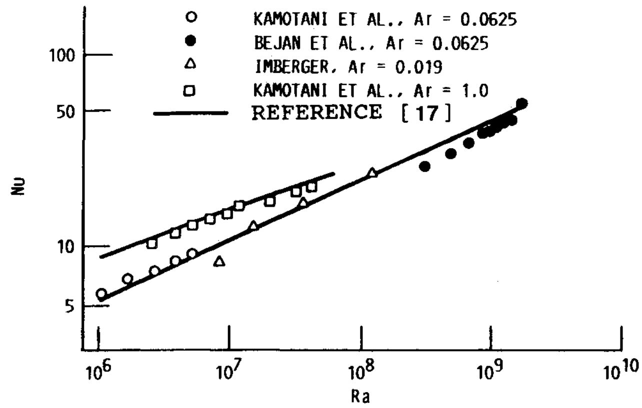

Fig. 5 Nusselt number in the boundary-layer regime.[17]

sets of data obtained by Kamotani et al., Bejan and Tien,[15] and Imberger.[16] The solid line represents a theoretical prediction using boundary-layer analysis by Bejan and Tien. As can be seen from experimental data, the Nusselt number becomes independent of the aspect ratio for very high Rayleigh numbers on the order of 10^8–10^9. However, the aspect ratio dependence of the Nusselt number is evident when the Rayleigh number reaches a more moderate range of 10^6–10^7. This indicates that a typical asymptotic analysis based on boundary-layer modeling (as appropriate to $Ra\rightarrow\infty$) is not capable of predicting the correct heat transfer coefficient in the moderately high Rayleigh number range (i.e., the aspect ratio dependence of Nu is not predicted). Bejan and Tien[15] also obtained approximate analytical solutions for Nusselt numbers corresponding to moderate Ra, using a different analysis, but many assumptions were made and the calculated Nusselt numbers tend to underpredict heat transfer results obtained experimentally for $\epsilon > 0.2$ (Ref. 14). What happens at moderately high Rayleigh numbers is mathematically similar to flow over a flat plate at moderate Reynolds numbers. It is well known that as $Re\rightarrow\infty$ the governing equations can be simplified to Prandtl boundary-layer equations that lead to approximate analytical solutions. However, the same simplification is not possible for moderate Reynolds number, and the original governing equations must be solved to obtain meaningful solutions.

A systematic way of treating moderate Rayleigh number problems was recently suggested in Ref. 17. The author utilized the results obtained for $Ra\rightarrow\infty$ in an analytical iteration procedure to upgrade and refine the asymptotic solutions valid for a wider range of Rayleigh numbers. The aspect ratio dependence of the Nusselt number observed by Kamotani et al.[14] for moderate Ra ranges was accurately predicted by the improved solutions (Fig. 5).

Concluding Remarks

It was shown that buoyancy-driven convection in a reduced-gravity environment can be of considerable importance in many processes. Two of the important natural convection parameters, the Grashof number (Gr) and the Rayleigh number (Ra), are both directly proportional to the magnitude of gravitational acceleration. The value of Gr for many typical gound-based materials-processing techniques is extremely large (Table 1). Mathematically, as $Gr\rightarrow\infty$, the governing equations display boundary-layer characteristics that result in a relatively simpler form of equations, and hence they may be solved analytically. In a reduced-gravity environment where the gravitational level is $g \sim 10^{-4}\, g_0$ to $10^{-2}\, g_0$, the Grashof number becomes smaller by a factor of 10^{-4}–10^{-2} than the values for the same process at $g = g_0$. The asymptotic boundary-layer solutions obtained for $Gr\rightarrow\infty$ are, in general, less accurate for moderate ranges of Grashof numbers and must be upgraded. A method for refining solutions was discussed.

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